Probabilistic Methods in Combinatorics

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Assignment 7

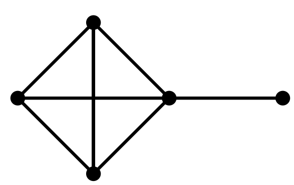
To solve for the Example class on 8th April. Submit the solution of Problem 2 by Sunday 6th April if you wish feedback on it. Some hints will be given on Friday 4th April.

The solution of each problem should be no longer than one page!

Starred problems are typically harder. Don't worry if you cannot solve them.

Problem 1. Show that $p_0 = n^{-1}$ is a weak threshold for the property that $G(n,p)$ contains K_3 (i.e. a triangle) as a subgraph.

Problem 2. What is a threshold probability function $p = p(n)$ for the occurrence of the graph below as a subgraph of the random graph $G(n,p)$?



Problem 3. For $n \geq 2$ and $p \in (0,1)$ consider the random 5-partite graph $G(n,p,5)$ defined as follows. The vertex set of $G(n,p,5)$ is a union of five disjoint independent sets $V_1, \ldots, V_5$, each of size n. Moreover, for $1 \leq i < j \leq 5$, each $(v_i, v_j) \in V_i \times V_j$ is an edge in $G(n,p,5)$ independently with probability p. Find a threshold probability function $p = p(n)$ for the occurence of K_5 as a subgraph of $G(n,p,5)$.

Problem 4*. Show there is a positive constant c such that the following holds. For any n reals $a_1, \ldots, a_n$ satisfying $\sum_{i=1}^n a_i^2 = 1$, if $(\varepsilon_1, \ldots, \varepsilon_n)$ is a $\{-1,1\}$-random vector obtained by choosing each ε_i randomly and independently with equal probability to be either -1 or 1, then

$$\Pr\left[\left|\sum_{i=1}^n \varepsilon_i a_i\right| \leq 1\right] \geq c.$$